DIVERSITY COMBINING USING CARRIER LOCK AND SIDEBAND LOCK TECHNIQUES*
PART II

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ABSTRACT. In this paper the design and performances of a diversity combining system using carrier Lock and Sideband Lock techniques have been discussed. The effects of noise have been studied with particular reference to the loop phase equation, signal to noise ratio, RMS phase error and limiting of the sum of signal and noise on the threshold value in a two channel predetection diversity combining system. The results of combining two modulated r.f. carriers bearing the same information are also stated. A calculation for the first order probability density function of the instantaneous phase for a carrier phase lock circuit is given. A scheme is suggested to correct the Doppler frequency shift of the received signal.

INTRODUCTION

In fading channels it is often found necessary to incorporate some kind of diversity for reliable communication. An essential requirement in diversity system is that the signals on the different independent channels be combined coherently. It should be clear that diversity combination requires locking of the carrier of each channel to its signal and also locking of the different signals on the various channels. Now it may so happen that the received signal power on a given channel may not be adequate to ensure satisfactory locking of its own demodulating carrier, while the total channel power may be large enough. An efficient locking technique should, therefore, make use, if possible, of the total channel power to establish lock (in AM one should utilise both carrier and sideband powers). We shall discuss the problem of diversity combination in section 2, with particular reference to the philosophy of inter and intra channel coherence to secure gain in the SNR of the combined output. Locking techniques in diversity reception with particular reference to (i) a pre-detection combiner and (ii) a post-detection combiner have also been discussed in this section (Chakrabarti, *et al.*, 1966).

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Section 3 is devoted to a discussion on the effects of noise on the loop phase equation, SNR at the output of the combiner, RMS phase perturbation of the locking loop and limiting of the received signal on the threshold value.

Experimental set-up and results of combining two modulated r.f. carriers, are given in section 4.

LOCKING TECHNIQUEL IN DIVERSITY RECEPTION

To satisfy the increasingly greater demand for long distance communication through radio waves is not only to provide additional communication channels but also to provide a reliable and efficient communication technique. Considerations of noise, interference and fading within the communication band and the time varying character of the propagation media call for the transmission of the same massage in a number of frequency channels simultaneously.

It is known that for certain communication applications it is desirable to use broad band signals in conjunction with coherent reception and that linear wide band systems (like frequency diversity of DSB, SSB) can be designed to operate reliably for low input signal to noise ratios. The argument can be stated as follows. In a linear system the output SNR equals the input SNR. Suppose that there are a number of such channels carrying the same signal. If the outputs of the different channels can be so combined that the signals add coherently and the weights used in combination are proportional to the signal to noise ratio of the corresponding channels then it can be shown that the SNR of the sum is equal to the sum of the SNR's (Kahn, L. R., 1954). This is optimum diversity combination. Even if the weights are unity, there is considerable gain in SNR, for now the output SNR is equal to $\Sigma s_k^2/\Sigma n_k^2$ where s_k and n_k are the signal and noise powers in the K-th channel. Two advantages arise. Firstly, the statistics of the combined noise and interference powers, which individually may have high ratios of the peak to r.m.s. assume after addition a fairly smooth character. Secondly, the output SNR shows considerable improvement over that of a single channel carrying an amount of power equal to the sum of the powers in the component channels. Such wide band systems thus have an inherent resistance to interference.

To establish Phase coherence inter and intra channels

If we have a frequency diversity of order two, there will be two r.f. signals carrying the same message although their input signal to noise ratio (SNR) will in general be different. Since the two inputs are at two different frequencies they will have to be brought back to the same frequency by appropriate local oscillators. Further since the r.f. phases after heterodyning will be different the modulation phase of the output of any coherent type detector will also be different. The phasing of the two r.f. signals to bring them into coherence can be done at r.f. (Brennan, D. G., 1959) and also at base band for linear demodulation systems.

Figs. (1) and (2) show the block representations to implement these techniques. For r.f. phase locking one would need a phase comparator or discriminator for measuring the difference in phase between the two r.f. signals and actuate the local oscillator in such a way as to reduce and if possible eliminate the phase difference between the two signals. For phasing at base band the demodulated outputs in the two channels can be compared by means of an audio frequency phase discriminator, the detected output of the discriminator controlling the r.f. phase of one of the channels.

The above mentioned differentially coherent technique enable one to bring the two inputs to a common reference. (It should be obvious that predetection phasing is the optimum technique if the demodulation is non-linear).

Locking through use of IF phase information

Fig. (1) shows the block diagram for a diversity combiner where relative phasing is accomplished at RF for a frequency diversity of order three. The intermediate frequency (i.f.) outputs of the three mixer stages are the same and the three i.f. outputs are brought in phase coherence before they are added and fed to a sideband lock receiver of the proper kind. Let us consider the input to the combiner is a frequency diversity of DSB signals of order three. Three frequency channels are separated using three frequency selective networks preceding the three mixer stages. Of the three local oscillators associated with the three mixer stages in the receiving system, two are of voltage controlled type and the third one along with its mixer is recognised as the reference chain and the corresponding i.f. output may be called as the reference i.f. output. The phase of the other two outputs are compared with it using two phase discriminators and d.c. controlling voltages are obtained in proportion to the phase departure between them. This d.c. controlling voltage is used to change the reactance offered by

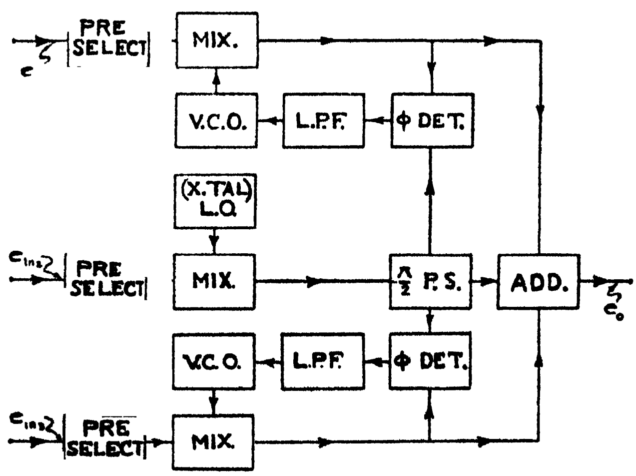

Fig. 1. Block diagram for coherent combination of the signals in a three channel frequency diversity system using r.f. phase information

a voltage sensitive diode which forms a frequency determining parameter in each of the VCO's. The d.c. controlling voltage is utilised in such a sense as to make the phase discrepancies between the i.f. outputs a minimum. It is seen that the reference i.f. output is given a phase shift of $\pi/2$ radians before multiplication. In the phase discriminator the low frequency output is proportional to the sine of the phase difference between its two inputs. Thus if the two i.f. outputs are in phase the product demodulator output is zero and the magnitude and polarity of this output depend on the magnitude of the phase difference between the two and whether one leads or lags the other.

The abovementioned technique is also suitable for combining diversity signals resulting from other types of modulations, linear as well as nonlinear.

To derive the expression for the control voltage in this case we note first that the IF output of the reference channel may be written to be

$$e_{Ref} = A \cos(\omega_{if}t + \phi_1 + \pi/2) \quad \dots \quad (1)$$

and that of the other channel

$$e_{oc} = B \cos(\omega_{if}t + \phi_2) \quad \dots \quad (2)$$

When these two IF outputs are multiplied one gets

$$e_d(t) = kAB \sin(\phi_1 - \phi_2) \quad \dots \quad (3)$$

as the controlling d.c. voltage.

Locking through use of modulation phase information

The combination of a diversity of signals can also be achieved using phase information from the detected modulation components. This technique is particularly suited for linear modulation systems. Fig. (2) shows the block diagram of such a Phasing Scheme. The input to such a combiner can be taken to be a frequency diversity DSB (since the modulation is linear) signal of order three. The three i.f. outputs in this case are fed to three DSB modulation receivers. It is known that the inphase channel output for such a receiving system corresponds to the modulation components. Before adding the three P channel outputs together it must be ascertained that they have been brought in phase coherence. Phase coherence between the modulation components is achieved by comparing the phase of the modulation output in the reference chain with those at the other two chains using two low frequency phase discriminators. The d.c. outputs of these phase detectors are used to control the phase of the VCO's in the respective chain. The phase detector output in this case is also proportional to sin ϕ_e where ϕ_e stands for the phase discrepancy between its two inputs. Each P channel output is fed to a wide band phase shifter associated with it. The outputs of each of the phase shifter are in phase quadrature with each other. So from the reference P channel two outputs are derived which are in phase quadrature and are applied to the two phase detectors as one of the two inputs.

The other input in each of the detector is obtained from the respective quadrature component of the outputs from the phase shifters in the other two chains.

To derive the expression for the control voltage in this case we note first that the P channel outputs in the two channels can be written as

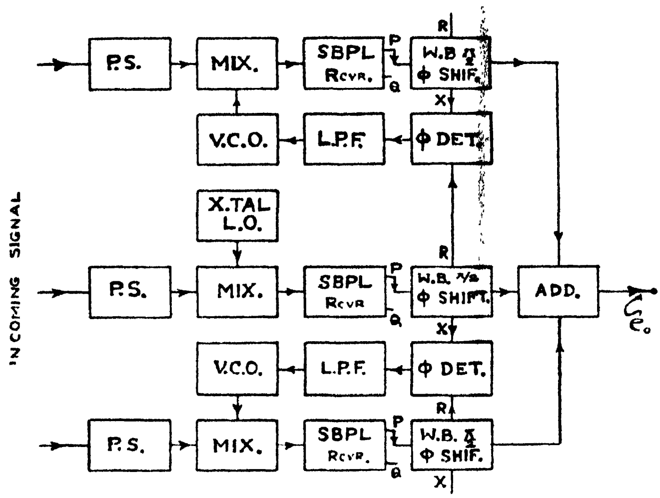

Fig. 2. Block diagram for coherent combination of the outputs of a three channel frequency diver- sity using modulation phase information.

$$e_{P_1}(t) = \Sigma(A_+ + A_-)\cos(\omega_s t + \phi_{A+} + \phi_{A-} - 2\psi_{0A}) \qquad \ldots \quad (4)$$

$$e_{P_2}(t) = \Sigma(B_+ + B_-)\cos(\omega_s t + \phi_{B+} + \phi_{B-} - 2\psi_{0B}) \qquad \ldots \quad (5)$$

If a phase shift of 90° is introduced between these two voltages and the outputs of the phase shifter are multiplied one gets

$$e_d(t) = k\Sigma(A_+ + A_-)(B_+ + B_-)\sin(\phi_{A+} + \phi_{A-} - 2\psi_{0A} - \phi_{B+} - \phi_{B-} + 2\psi_{0B})$$

as the slowly varying d.c. voltage.

It is obvious that this expression has a very close similarity to equation (3) and all the remarks made about the latter obviously apply.

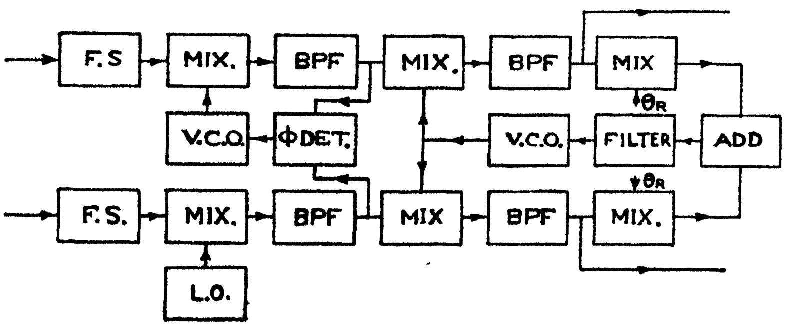

Fig. 3. Block diagram of a scheme to correct Doppler frequency shift.

The techniques mentioned above are meant for correcting phase perturbations between the signals in the different channels. There may sometimes be some disturbances common to all the channels e.g. Doppler shift. In such cases it is desirable to correct these common disturbances by means of a control circuit operated by the average value of a sum of the instantaneous phases in the different channels. Schematic diagram of such a control circuit is given in Fig. (3).

PHASE EQUATION AND SIGNAL TO NOISE RATIO IN A TWO-CHANNEL PREDETECTION COMBINER DIVERSITY SYSTEM

Let us consider the system of Fig. (1). If the inputs (1) and (2) are represented by

$$e_{i_1}(t) = \Sigma A_u \cos(\omega_0 t + ut + \phi_u) + n_1 \cos(\omega_0 t + \phi_{n1}) \quad \dots (7)$$

and

$$e_{i_2}(t) = \Sigma A_v \sin(\omega_0 t + vt + \phi_v) + n_2 \sin(\omega_0 t + \phi_{n2}) \quad \dots (8)$$

the output of the product modulator will be

$$e_d(t) = \Sigma\, \Sigma A_u A_v \sin(vt - ut + \phi_v - \phi_u) + \Sigma A_k n_2 \sin(\phi_{n2} - \omega_k t - \phi_k) \quad \dots (9)$$

In the particular case when the signals are of the CW type the expression for the output may be written more simply as

$$e_d(t) = [A_1 A_2 \sin(\phi_1 - \phi_2) + n_1 A_2 \sin(\phi_2 - \phi_{n1})$$

$$+ n_2 A_1 \sin(\phi_{n2} - \phi_1) + n_1 n_2 \sin(\phi_{n1} - \phi_{n2}) \quad (10)$$

It will be seen that this output contains a term proportional to the sine of the difference between the phases of the two signals, a term representing intermodulation between the signal and noise and the third term representing intermodulation between the noise components.

In general the difference in phase $\phi_1 - \phi_2$ will be a slowly varying function of time and one can therefore use a low pass filter of narrow bandwidth to accept only the slowly varying components. The detected output thus filtered has been contaminated with noise and intermodulation components in the band of this filter.

The filtered output as shown in the diagram controls the instantaneous frequency of the VCO. The phase equation that results is

$$\frac{d\phi}{dt} = Kf(p) \sin(\phi_1 - \phi_2 - \psi_0) \quad \dots (11)$$

If the effective signal to noise ratio in the closed loop bandwidth of the controlled loop be adequate then it is reasonable to assume that the magnitude of the phase $(\phi_1-\phi_2-\psi_0)$) will be small and one can write the phase equation in the linearised form as

$$\frac{d\phi}{dt}=Kf(p)(\phi_1-\phi_2-\psi_0) \qquad \ldots \quad (12)$$

It should be mentioned that although there will be steady state phase error the phase difference $\phi_1-\phi_2$ will be perturbed by the noise terms present. The summed output can be written as

$$e_T(t)=[A_1 \exp j(\omega_0 t+\phi_1-\psi_0)+A_2 \exp j(\omega_0 t+\phi_1-\psi_0-\phi_e)$$
$$+n_1 \exp j(\omega_0 t+\phi_{1n})+n_2 \exp j(\omega_0 t+\phi_{2n})] \qquad \ldots \quad (13)$$

where ϕ_e is the phase error. The amplitude of the instantaneous signal will thus be

$$S=[A_1^2+A_2^2+2A_1A_2 \cos\phi_e]^{\frac{1}{2}} \qquad \ldots \quad (14)$$

If the noises appearing in the two channels have a correlation coefficient ρ then the total noise voltage will be given by

$$N=[n_1^2+n_2^2+2\rho n_1 n_2]^{\frac{1}{2}} \qquad \ldots \quad (15)$$

In the case of three channels the corresponding signal and noise outputs are

$$S=[A_1^2+A_2^2+A_3^2+2(A_1A_2 \cos\phi_{12}+A_2A_3 \cos\phi_{23}+A_3A_1 \cos\phi_{31})]^{\frac{1}{2}} \qquad (16)$$

$$N=[n_1^2+n_2^2+n_3^2+2(\rho_{12}n_1n_2+\rho_{23}n_2n_3+\rho_{31}n_3n_1)]^{\frac{1}{2}} \qquad \ldots \quad (17)$$

Effect of limiting the sum of signal and noise on the threshold value

Threshold : Threshold SNR is defined as the value of the signal to noise ratio at which the rate of change of output SNR with the input SNR shows an abrupt break. Threshold phenomenon is due to nonlinear processes occurring in demodulation which cause through intermodulation a rise in the value of the noise appearing in the output.

In CPL threshold SNR (power) in the effective bandwidth of the system is about 5 or 7 db.

Limiting : It is known that if a sum of two signals is limited and the limited signal is passed through a bandpass filter there is a general suppression of the weaker component. The amount of this suppression depends on the amplitude ratio and the difference in frequency. This result can be carried over to the analysis of the situation when one of the signals is replaced by a random noise.

The probability of the noise amplitude being above a certain value C, is given by

$$\frac{2}{\sqrt{\pi}}\int_0^C e^{-x^2/2N_0}\cdot dx\cdot\frac{1+erf\sqrt{R}}{2}\,.$$

This shows that at a signal to noise ratio of R the favourable period is more than the unfavourable by a factor equal to $\frac{1+erf\sqrt{R}}{1-erf\sqrt{R}}$. If we assume that the average suppression during the favourable period is the same as that in the unfavourable period, the average SNR at the output will show an apparent improvement.

This improvement in SNR is however of no consequence in binary PSK reception if the error probability is less than the minimum desired value before limiting. This is so because the error probability is determined essentially by the condition that the value of the in phase component of noise be less than a certain maximum in order that the number of times the r.f. phase is modified by the noise is less than a prescribed minimum. Limiting filtering cannot alter this condition and therefore has no effect on the threshold value.

R.M.S. Phase Erorr : It is of interest to form an estimate of the perturbation produced by noise in the feedback loop. In fact the effective signal strength is always found to be multiplied by the term $\cos\phi_e$ where ϕ_e is the phase error due to noise. Further there may be instant of time when the phase disturbance due to noise might cause loss of synchronisation.

An examination of the noise terms in the loop phase equation (Refer Eq. 9) shows that the output noise is in general a nonlinear function of the input noise except when the signal to noise ratio is high. One of the terms in this equation can be considered to have arisen due to phase modulation of the noise in the band by a voltage proportional to the carrier phase. It is therefore expected that the spectrum of this intermodulation will spread over the band equal to the sum of the noise band and the band of the carrier phase modulation. If the latter is small the spectral character of this intermodulation term can be considered to be the same as that of the input noise. The effect of another term which is due to intermodulation between noise components will also have to be considered unless the signal to noise ratio is adequate.

To analyse the phase equation it is convenient to lump all the noise terms together as a single noise term having approximately the same spectral character as the input noise. However, its magnitude increases more sharply than the input. The equivalent equation is

$$\frac{d\phi}{dt} = \Omega - Kf(p)[A\sin\phi + N] \tag{18}$$

To find the probability distribution of the loop phase the above equivalent equation must first be converted into corresponding equation for the probability density, W. This equation when $f(p) = 1$ and $\Omega = 0$, is known to be given by

$$\frac{\partial W}{\partial t} = \frac{N_0K^2}{4} \frac{\partial^2 W}{\partial\theta^2} + K\frac{\partial}{\partial\theta}(A\sin\theta\cdot W) \tag{19}$$

where N_0 is the noise power.

It is easy to verify that the solution to this equation in the steady state $\frac{\partial W}{\partial t} = 0$ is

$$W(\theta) = \frac{\exp(a\cos\theta)}{2\pi I_0(a)}, \quad \text{where} \quad a = \frac{4A}{KN_0}.$$

An approximate solution of the derived equation for a general $f(p)$ can be found by first linearising the equation and solving it, and replacing the first power of θ by $\sin\theta$ and θ^2 by $2(1-\cos\theta)$ in the solution. Once the probability density is determined, the square phase error and the mean value of $\cos\phi$ can be readily found by evaluating the integrals

$$\overline{\phi^2} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \phi^2 W(\phi)\cdot d\phi \qquad \ldots (20)$$

$$\overline{\cos\phi} = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos\phi\, W(\phi)\cdot d\phi \qquad \ldots (21)$$

In the particular cases when $f(p) = 1$ or $f(p) = \frac{1}{1+p\tau}$, the value of $\cos\phi_e$ is $\frac{I_1(a)}{I_0(a)}$, where $a = \frac{A^2}{\text{Noise density} \times \text{noise bandwidth}}$.

EXPERIMENTAL SET-UP, RESULTS AND DISCUSSIONS

Fig. 4. shows the block diagram of the experimental arrangements for the coherent combination of two r.f. carriers at frequencies 1.55 mc/s and 1.85 mc/s

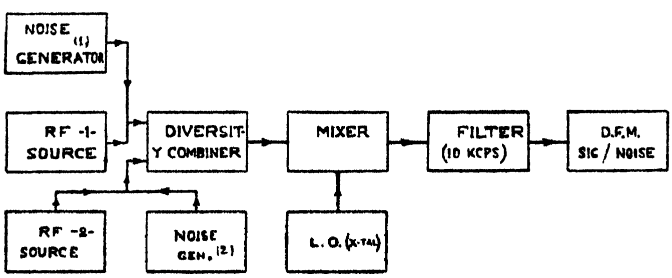

Fig. 4. Block diagram of the experimental set-up for coherent combinations of two r.f. carriers.

respectively. The frequencies of the oscillators in the respective mixers are higher than the incoming carriers by 500 Kc/s. One of the oscillator is a V.C.O. and a V-33 type varactor is used in the frequency determining circuit. The voltage sensitivity of the VCO is 15 Kc/s per volt. The reference oscillator is that of a Clapp's type. A crystal controlled oscillator may preferably be used as a reference one.

In the design of the phase control loop, consideration has been given to the correlation time, maximum fading rate and noise bandwidth.

For the purpose of simulation of multiplicative noise caused by the propagation medium, one has to take into account, besides a nearly steady Doppler shift due to a regular drift, (i) flat fading, where the band of interest is subject, as a whole, to variations of amplitude and phase; (ii) selective fading where the fluctuations of amplitude and phase of different groups of frequencies in the band are relatively independent and (iii) multipath phenomena where a large number of identifiable paths having distinct time delays and amplitude -phase fluctuations contribute to the total received signal. For the first case one may multiply the input with the sum of a carrier and a narrowband noise around it. For the second the input is distributed to several bandpass channels and amplitude-phase fluctuations are introduced from narrowband noise sources. For multipath simulation provision is made for fixed and variable time delays and amplitude-phase variations of the different outputs from the tapped delay line. The bandwidth of the noise sources will have to be consistent with the fading rate both in amplitude and phase in the frequency band of interest. (The rms fading rate at H.F. determined essentially by the random velocity distribution of the scattering sources is known to be about $10/\lambda$ c/s where λ is in meters)

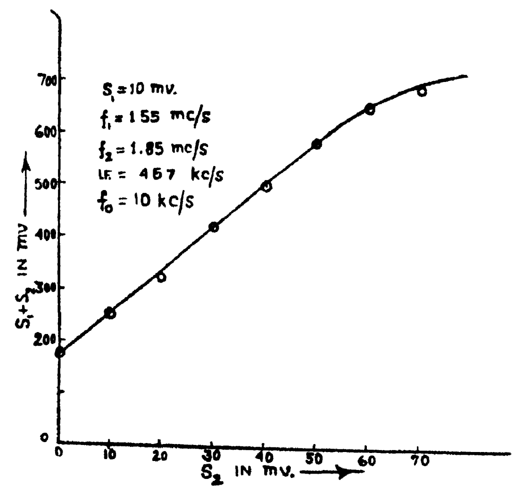

Fig. 5. Figure shows the variation of the combined output after coherent combination of two r.f. carriers with the change in level of either of the two carriers, the level of the other is held constant.

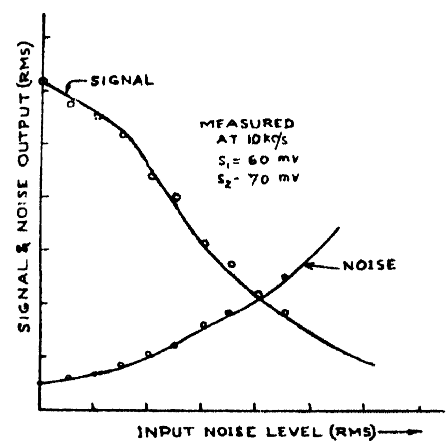

Fig. 6. Figure shows the variation of the signal and noise output in the combined output with the change in noise levels at the input to the combiner, the levels of the carriers are held constant.

The experimental result presented in Fig. (5) shows that the combined output due to two R.F. inputs increases linearly over a certain range of the inputs. The minimum acceptable value is determined by the total noise while the maximum value is controlled by the system non-linearity. From the results shown in Fig. (6) we note that as the input SNR deteriorates there is a reduction in the summed output. This is due as mentioned in Sec. 3 to partial incoherence at the summing point due to inadequacy of the phase following loop. The noise output at the summing point shows a linear variation upto certain level of input noise after which there is a marked increase in he slope. This is due to the threshold phenomenon setting in as discussed in Sec. 3.

It should be emphasised that for the success of any diversity system a knowledge of such medium properties as correlation bandwidth in respect of envelope and phase, amount of correlation between the signal as well as noise fluctuations in the component channels and the fading spectrum is essential. Such data requiring analysis of observations over a long period, unfortunately, are not readily obtainable. This problem of channel estimation will be considered in a future communication.

A C K N O W L E D G E M E T S S

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A P P E N D I X

First order probability density function of instantaneous phase for a CPL circuit

To find the probability density of the instantaneous phase in a phase locking loop one may first develop the loop dynamical equation into the corresponding Fokker Planck equation. In the case when the loop filter is given by

$$f(p) = \frac{1}{1+p\tau}\,.$$

the loep equation becomes

$$(1+p\tau)\frac{d\theta}{dt}\cdot +KA\sin\theta = K(\Omega_1 - n)$$

where

$$\Omega_1 = \frac{1}{K}[(1+p\tau)\Omega] \qquad \ldots \text{(A1)}$$

Now the frequency deviation Ω may be a random variable with or without a steady component. If it has no steady component R.H.S. of Al may be replaced by a random variable having a power density determined by Ω_1 and n. The Fokker Planck equation for the steady state probability distribution in such a case can be written as

$$0 = -\frac{\partial}{\partial\theta}(x_1 W) + \frac{\partial}{\partial x_1}\left[\frac{(\theta + KA\sin\theta)}{\tau^2} W\right] + \frac{K^2N_0}{4\tau^2}\cdot\frac{\partial^2 W}{\partial x_1^2} \quad \ldots \quad \text{(A2)}$$

where $x_1 = \dot{\theta}$.

It can be readily verified that the solution of the above equation is given by

$$W(\theta, \dot{\theta}) = c\exp\left((a\cos\theta + \frac{\beta}{2}\dot{\theta}^2\right) \quad \ldots \quad \text{(A3)}$$

where
$$a = \frac{4A}{K^2N_0}\ ; \quad \beta = -\frac{4\tau}{K^2N_0}$$

Now if
$$f(p) = \frac{1+p\tau_0}{1+p\tau}$$

$$\frac{d\theta}{dt} = \Omega - Kf(p)(A\sin\theta + n)$$

or,
$$p\theta = \Omega - K\frac{1+p\tau_0}{1+p\tau}\cdot(A\sin\theta + n)$$

Writing $(1+p\tau_0)x = \theta$ one gets

$$px_1 = \frac{\Omega}{1+p\tau_0} - \frac{K}{1+p\tau}(A\sin\theta + n) \quad \ldots \quad \text{(A4)}$$

or,
$$(1+p\tau)\frac{\partial x_1}{\partial t} + KA\sin\theta = K\left[\frac{\Omega}{K}\frac{1+p\tau}{1+p\tau_0} - n\right]$$

If $\Omega = 0$, the Fokker Planck equation in the steady state is

$$x_1\frac{\partial W}{\partial x_1} = \frac{\partial}{\partial x_1}\left[\frac{(x_1 + KA\sin\theta)}{\tau}\cdot W\right] + \frac{K^2N_0}{4\tau^2}\cdot\frac{\partial^2 W}{\partial x_1^2} \quad \ldots \quad \text{(A5)}$$

where as usual $x_1 = \dot{\theta}$

The solution to the equation obtained by linearising the *F-P* equation can be found by noting that the solution in the linear case is given by

$$W(\theta, \dot{\theta}) = c\exp\left(\frac{\alpha\theta^2}{2} + \frac{\beta x_1^2}{2}\right) \quad \ldots \quad \text{(A6)}$$

where
$$\alpha = \frac{\text{Signal Power}}{\text{Noise power in the Noise band width}}$$

and
$$\beta = \frac{\alpha}{4\pi^2}\frac{1}{\text{Second moment of the noise spectrum}}$$